Scale resolution in the information lattice

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SciPost Physics **13**, 080 (2022); PRX Quantum **5**, 020352 (2024); PRL **134**, 190401 (2025); arXiv:2510.26696; arXiv:2512:20793

Overview

The **information lattice** provides a decomposition of the total information $I(\rho)$ in a quantum state ρ into scale- and position-resolved **local information** i_n^ℓ

$$I(\rho) = \sum_{n,\ell} i_n^\ell$$

The local information i_n^ℓ is **locally conserved** on the information lattice (spanning the values of n and ℓ)

its structure provides a **universal characterisation** of states and

its dynamics the basis for **time evolution** algorithms like LITE (local information time evolution)

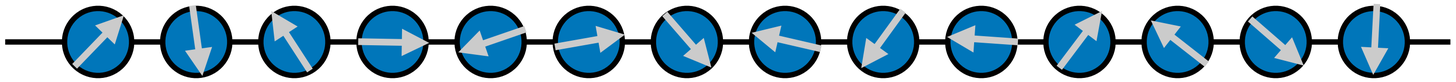
[Klein Kvorning, Herviou, JHB SciPost Physics (2022); Artiaco, Fleckenstein, Aceituno Chávez, Klein Kvorning, JHB PRX Quantum 2024]

How much information is there in the state ρ of L qubits?

$$I(\rho) = \log_2(\dim\rho) - S(\rho) = L + \operatorname{tr}\rho\log\rho$$

$$= L + \min_{\{\Lambda_y\}} \sum_y \mathsf{Tr}\left(\Lambda_y\rho\right)\,\log\left[\mathsf{Tr}\left(\Lambda_y\rho\right)\right]$$

Example: product state has L bits corresponding to measuring the direction of each qubit



Example: infinite temperature state $\rho_\infty = \mathbb{1}/\dim(\rho)$ has no information

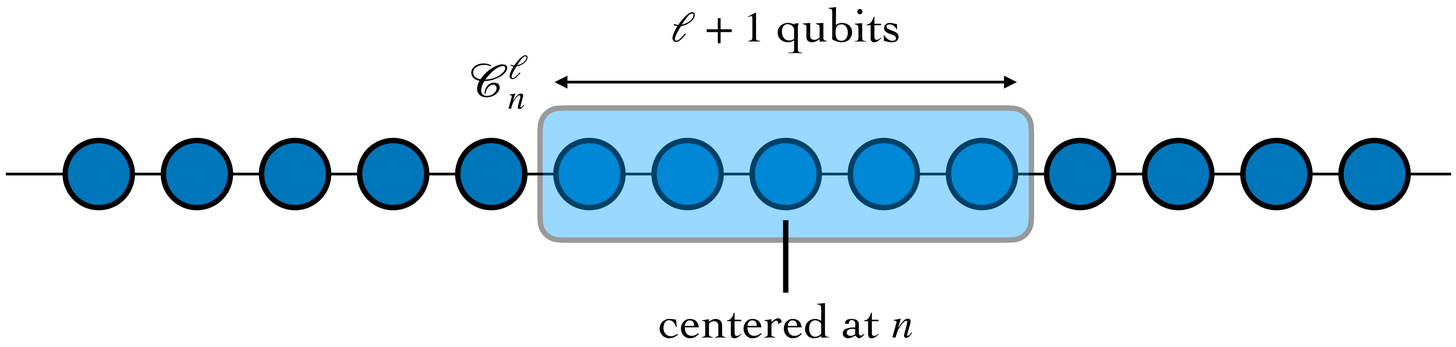
Example: singlet $|\psi\rangle = \frac{1}{\sqrt{2}}\left(|01\rangle - |10\rangle\right)$ has two bits corresponding to, for example, measuring $-XX$ and $-ZZ$

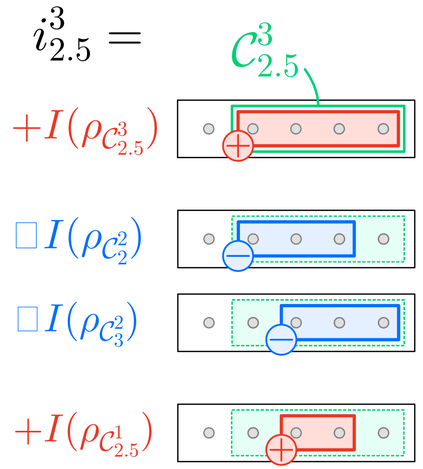
Example: stabiliser state $Q_i|\psi\rangle = |\psi\rangle$ with L commuting stabilisers $[Q_i, Q_j] = 0$

[Klein Kvorning, Herviou, JHB SciPost Physics (2022); Artiaco, Fleckenstein, Aceituno Chávez, Klein Kvorning, JHB PRX Quantum 2024]

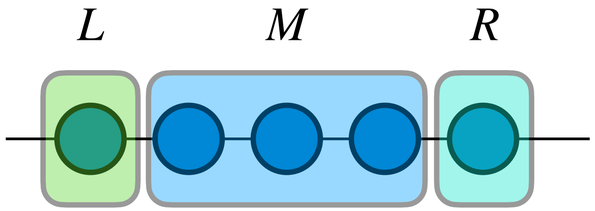
Local information

The local information i_n^ℓ is the information in a subsystem $\mathscr{C}_n^\ell$ that is not contained in any smaller subsystem





$$i_n^\ell = I_n^\ell - I_{n-1/2}^{\ell-1} - I_{n+1/2}^{\ell-1} + I_n^{\ell-2} \geq 0$$

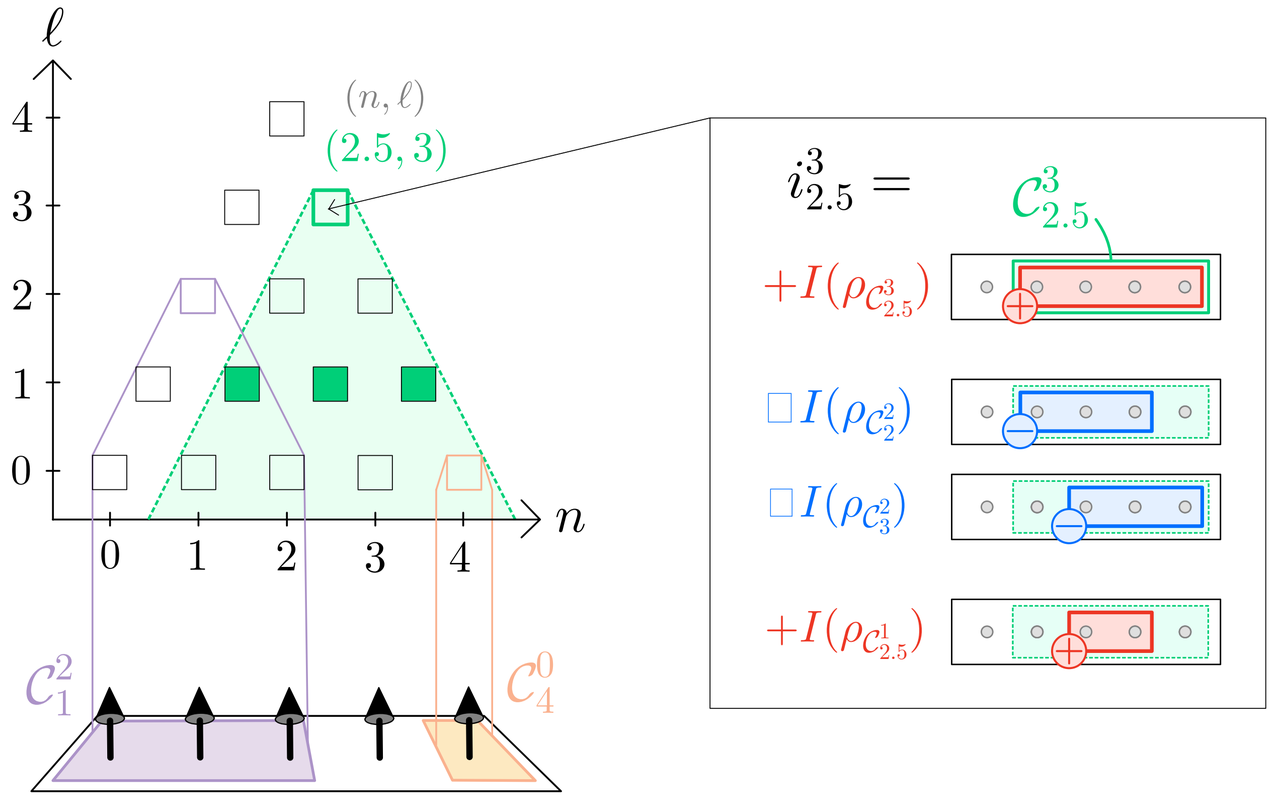


$$i(LMR) = I(LMR) - I(LM) - I(MR) + I(M) = I(L:R|M)$$

[Klein Kvorning, Herviou, JHB SciPost Physics (2022); Artiaco, Fleckenstein, Aceituno Chávez, Klein Kvorning, JHB PRX Quantum 2024]

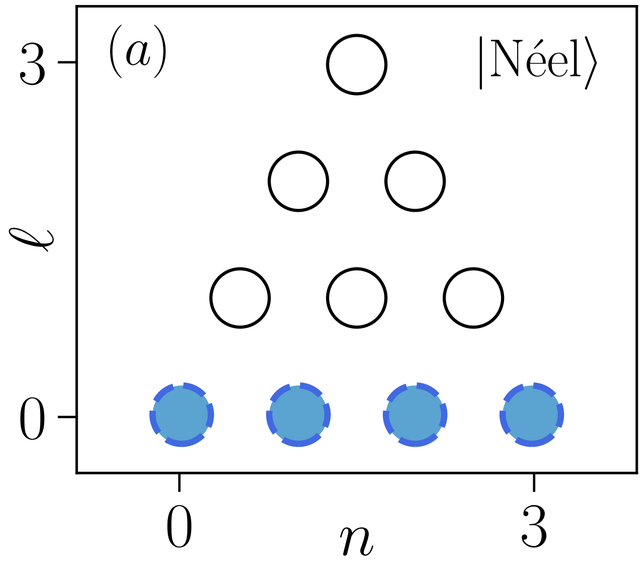
The information lattice is pictorially visualised as a triangle

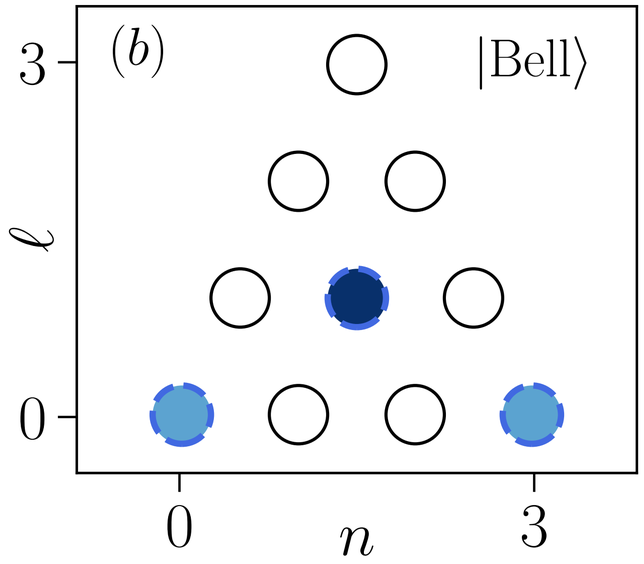
Each point (n, ℓ) corresponds to a subsystem $\mathscr{C}_n^\ell$ and the value at the point is the corresponding local information i_n^ℓ

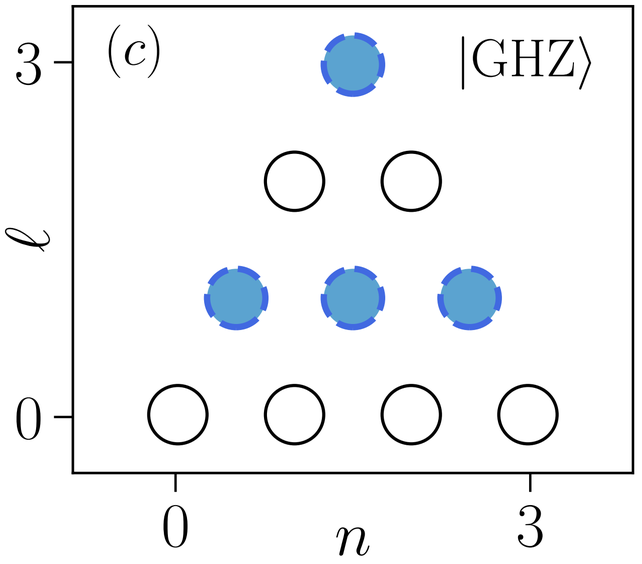


[Klein Kvorning, Herviou, JHB SciPost Physics (2022); Artiaco, Fleckenstein, Aceituno Chávez, Klein Kvorning, JHB PRX Quantum 2024]

Information lattice for stabiliser states has integer information







$\mathcal{G}_{\mathrm{GHZ}}$

$\ell=3$	$\ell=2$
$XXXX$	$ZIZI$
$XYYX$	$IZIZ$
$YXYX$	$\ell=1$
$YYXX$	$ZZII$
$XXYY$	$IZZI$
$XYXY$	$IIZZ$
$YXXY$	$\ell=0$
$YYYY$	$\emptyset$
$ZZZZ$	
$ZIIZ$	

$$|\text{Néel}\rangle = |0101\rangle \qquad |\text{Bell}\rangle = \frac{1}{\sqrt{2}}|0\rangle \otimes (|01\rangle + |10\rangle) \otimes |0\rangle \qquad |\text{GHZ}\rangle = \frac{1}{\sqrt{2}}(|0000\rangle + |1111\rangle)$$

$$IXXI\,|\text{Bell}\rangle = |\text{Bell}\rangle$$

$$IZZI\,|\text{Bell}\rangle = |\text{Bell}\rangle$$

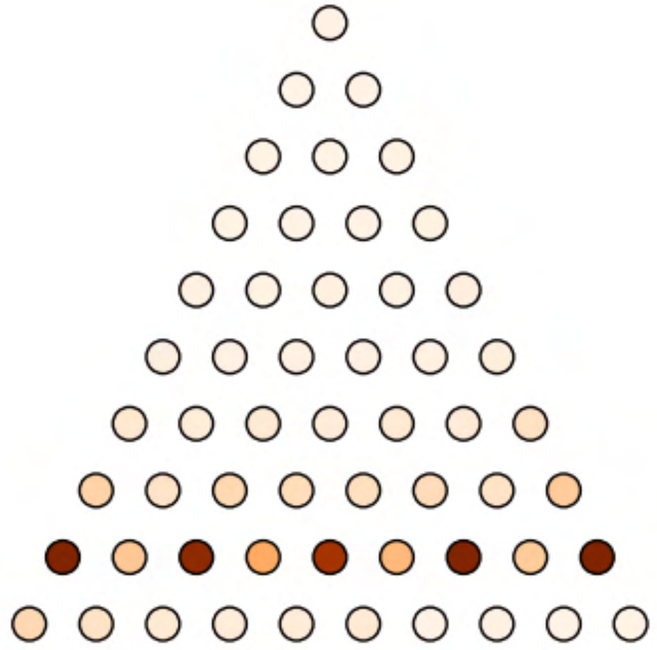
$i_n^\ell = |\mathcal{G}_n^\ell| - |\mathcal{G}_{n-1/2}^{\ell-1}| - |\mathcal{G}_{n+1/2}^{\ell-1}| + |\mathcal{G}_n^{\ell-2}| = \in \{0,1,2\}$ where $\mathcal{G}_n^\ell$ is the stabilizer subgroup of subsystem $\mathcal{C}_n^\ell$

[Bilinskaya, Martínez, Ghosh, Klein Kvorning, Artiaco, JHB, arXiv:2510.26696]

The information lattice for eigenstates of the XXZ model with random field

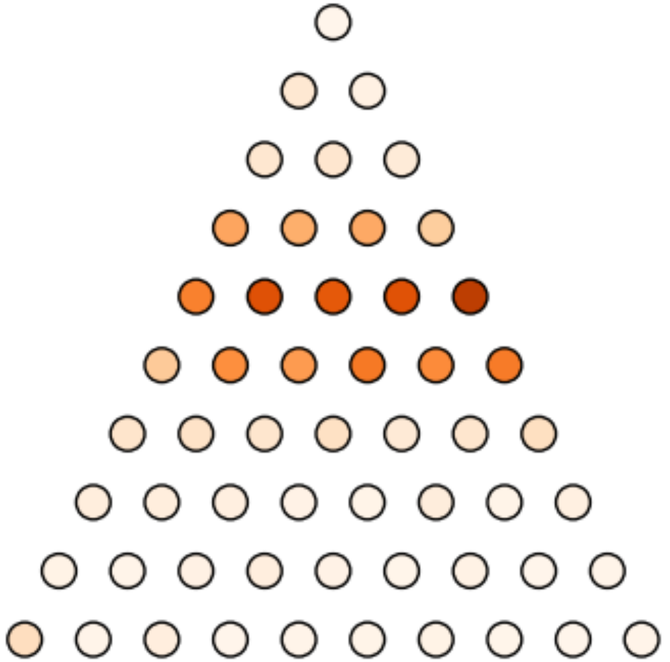


$|E_0\rangle$



Area Law

$S \sim L_A^{d-1}$



Volume Law

Volume law $S \sim l_A^d$



$|E_M\rangle$

Total information on a given scale is characteristic of the state

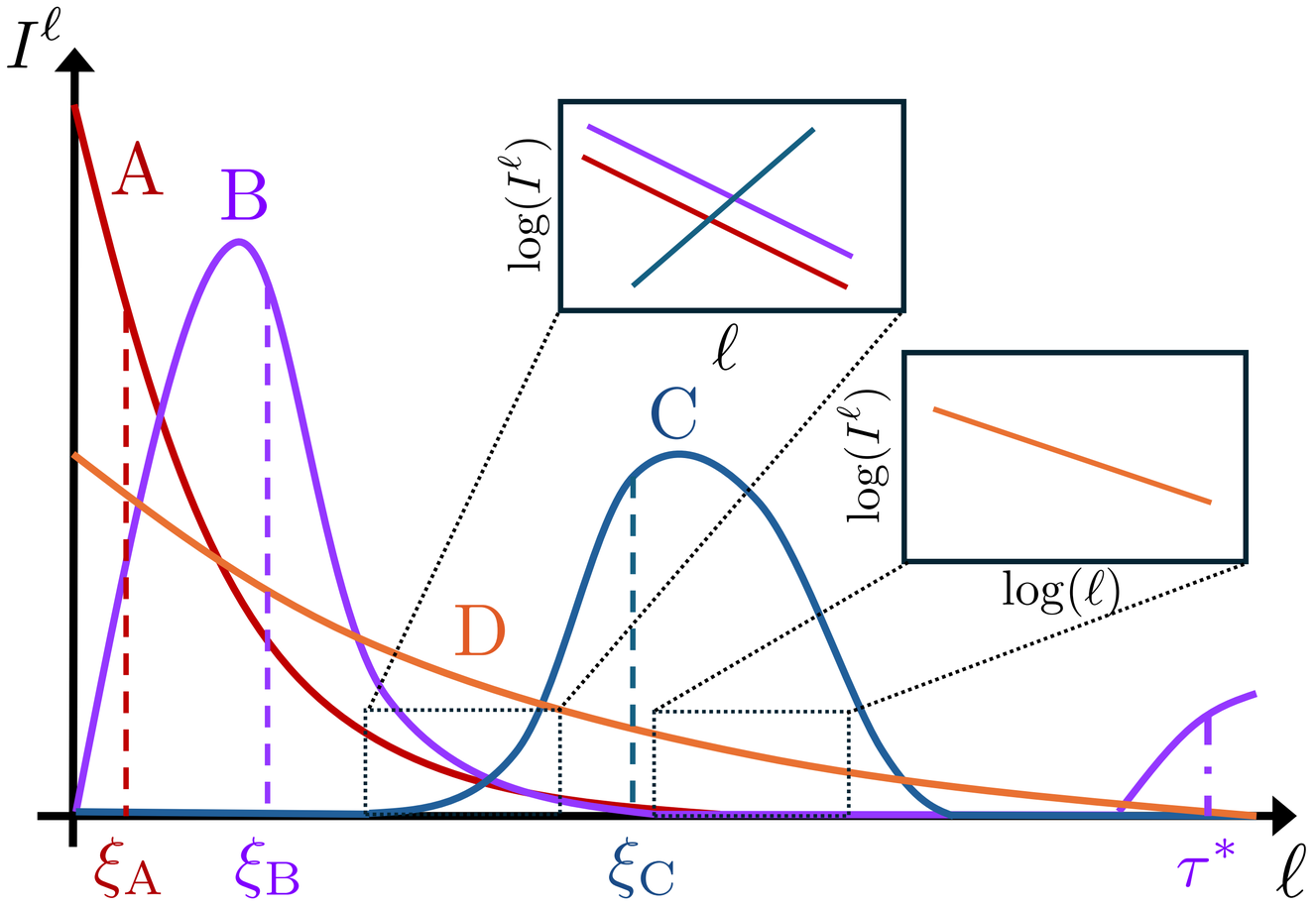
The total information $I^\ell = \sum_n i_n^\ell$ on scale ℓ

A—localized state

B—topological localized state

C—fully ergodic state (ETH $T = \infty$)

D—critical state



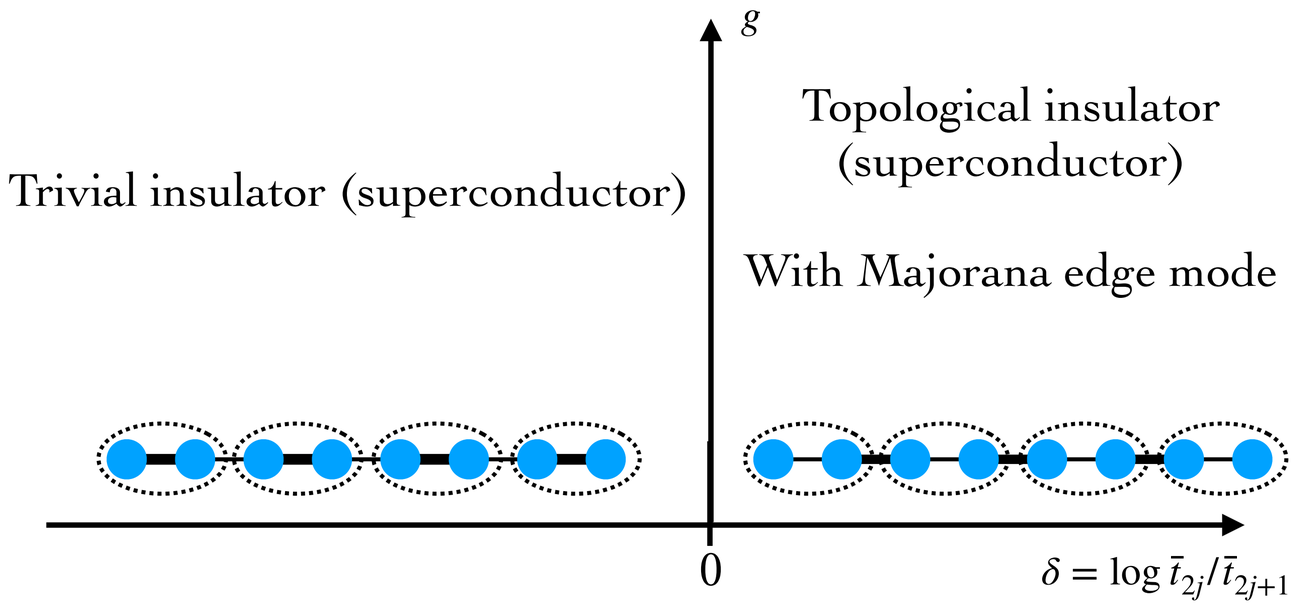
[Artiaco, Klein-Kvorning, Aceituno Chávez, Herviou, JHB, PRL 2025 (arXiv:2410.10971)]

Example: the various phases of the disordered Ising-Majorana model

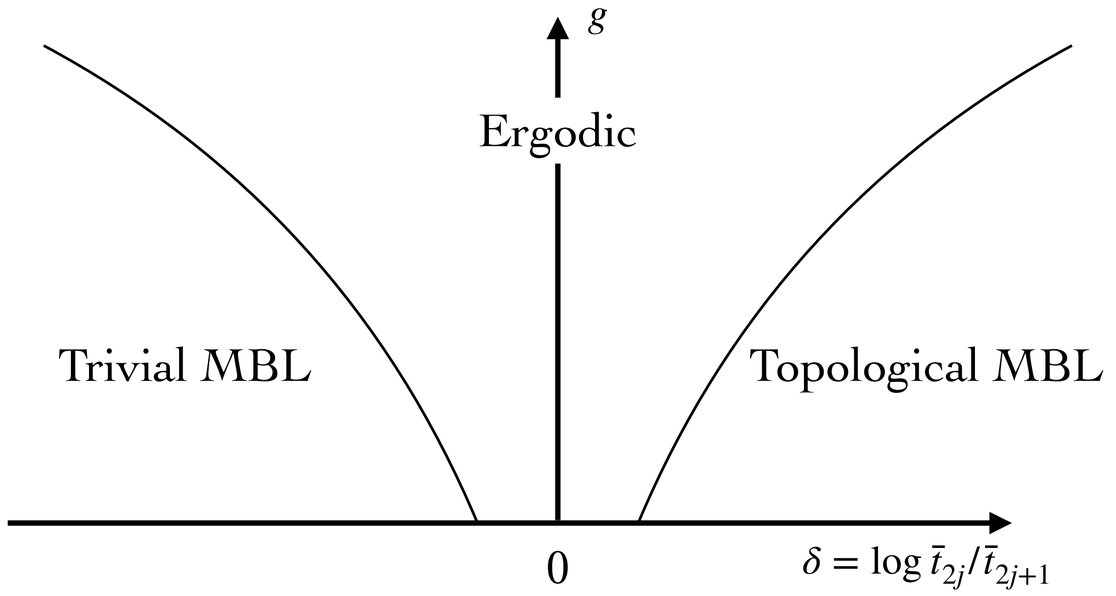
$$H = -i\sum_{j=1}^{2L-1} t_j\gamma_j\gamma_{j+1} + g\sum_{j=1}^{2L-3}\gamma_j\gamma_{j+1}\gamma_{j+2}\gamma_{j+3} \qquad t_{2j-1}\in[0,e^{-\delta/2}] \qquad t_{2j}\in[0,e^{\delta/2}]$$

$$c_i = \gamma_{2i-1} + i\gamma_{2i} \qquad \gamma_i^\dagger = \gamma_i$$
$$c_i^\dagger = \gamma_{2i-1} - i\gamma_{2i} \qquad \{\gamma_i,\gamma_j\} = 2\delta_{i,j}$$

Ground state phase diagram



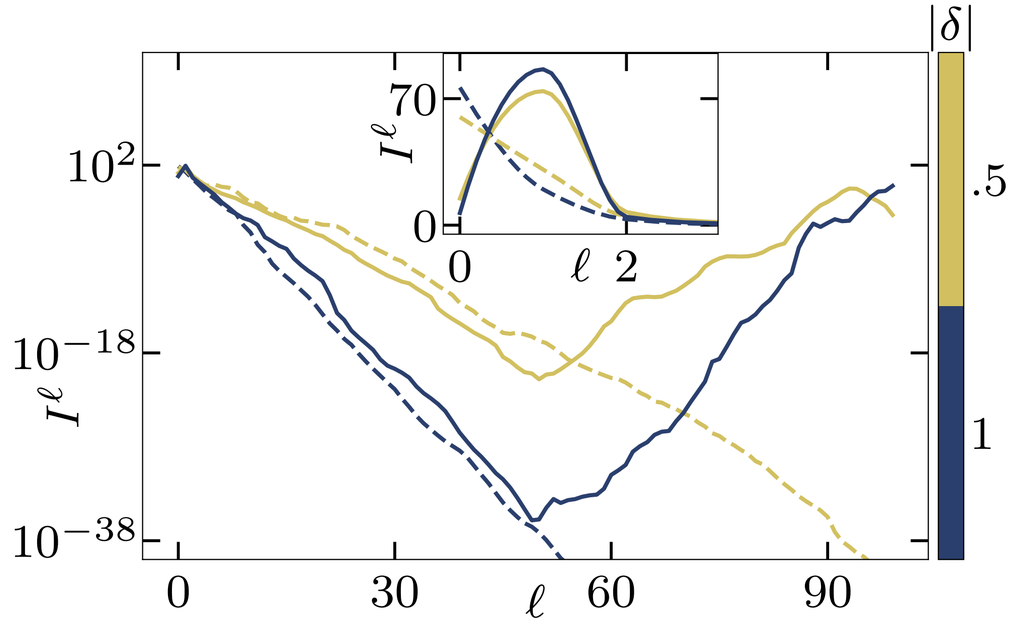
Excited state many-body localisation (MBL) phase diagram

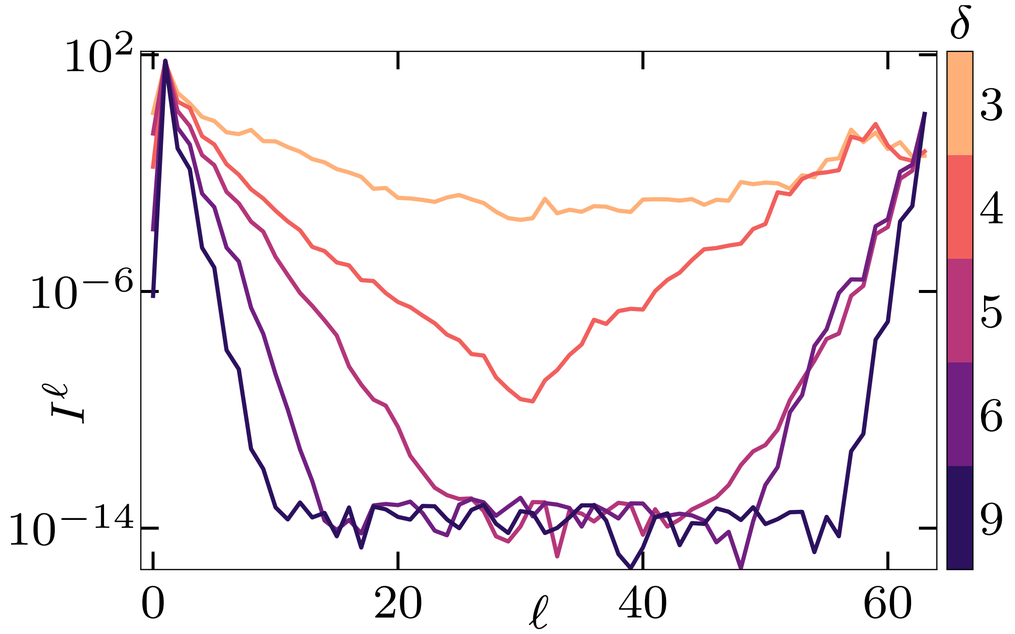


[Artiaco, Klein-Kvorning, Aceituno Chávez, Herviou, JHB, PRL 2025 (arXiv:2410.10971)]

Example: the various phases of the Ising-Majorana model

$$H = -i\sum_{j=1}^{2L-1} t_j \gamma_j \gamma_{j+1} + g\sum_{j=1}^{2L-3} \gamma_j \gamma_{j+1} \gamma_{j+2} \gamma_{j+3} \qquad t_{2j-1} \in [0, e^{-\delta/2}] \qquad t_{2j} \in [0, e^{\delta/2}]$$





For $g = 0$ noninteracting Kitaev model with two phases:

i) trivial superconductor / insulator

ii) topological superconductor with Majorana edge modes

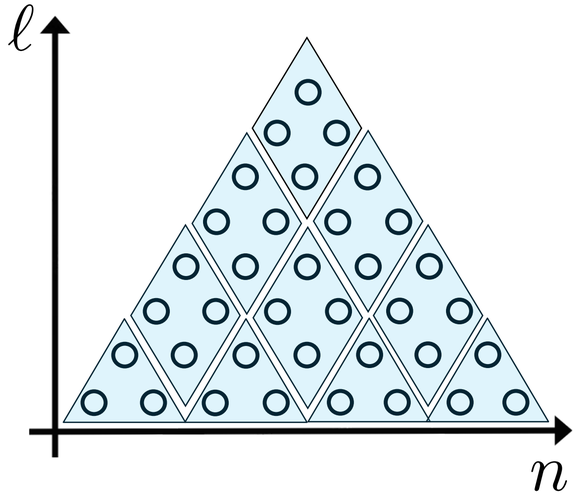
Interactions $g \neq 0$ have same phases in ground state

[Artiaco, Klein-Kvorning, Aceituno Chávez, Herviou, JHB, PRL 2025 (arXiv:2410.10971)]

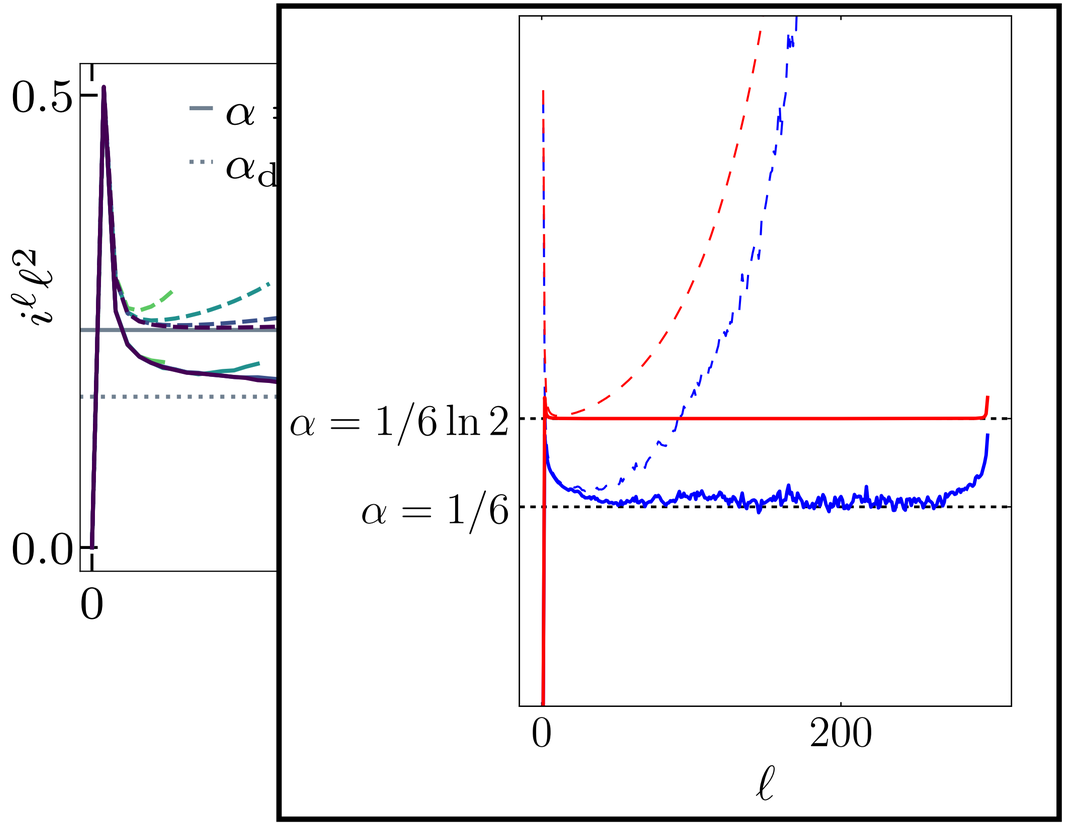
Example: the various phases of the Ising-Majorana model

$$H = -i\sum_{j=1}^{2L-1} t_j \gamma_j \gamma_{j+1} + g\sum_{j=1}^{2L-3} \gamma_j \gamma_{j+1} \gamma_{j+2} \gamma_{j+3} \qquad t_{2j-1} \in [0, e^{-\delta/2}] \qquad t_{2j} \in [0, e^{\delta/2}]$$

For $g = 0$ and $\delta = 0$ we are at a critical point in between the normal superconductor and the topological superconductor



Scale invariance implies $i^{\ell} \to i^{\ell/\ell^*} = (\ell^*)^2 i^{\ell} \Rightarrow \quad i^{\ell} = \dfrac{\alpha}{\ell^2}$



[Artiaco, Klein-Kvorning, Aceituno Chávez, Herviou, JHB, PRL 2025 (arXiv:2410.10971)]

Example: the various phases of the Ising-Majorana model

$$H = -i\sum_{j=1}^{2L-1} t_j \gamma_j \gamma_{j+1} + g\sum_{j=1}^{2L-3} \gamma_j \gamma_{j+1} \gamma_{j+2} \gamma_{j+3}$$

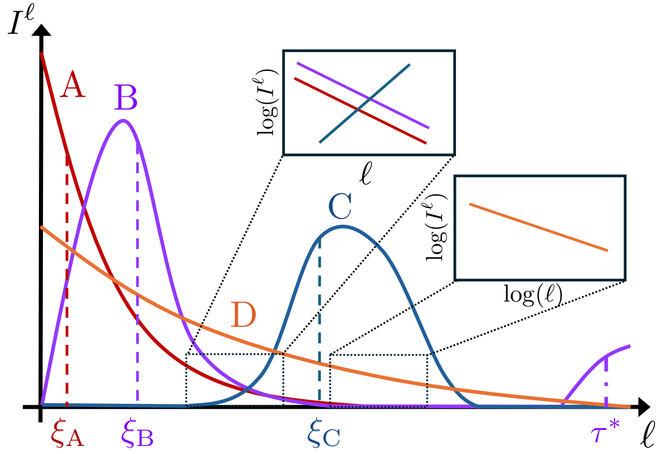
$$t_{2j-1} \in [0, e^{-\delta/2}] \qquad t_{2j} \in [0, e^{\delta/2}]$$

For $g > 0$ and midspectrum states there are three types of states (in finite systems):

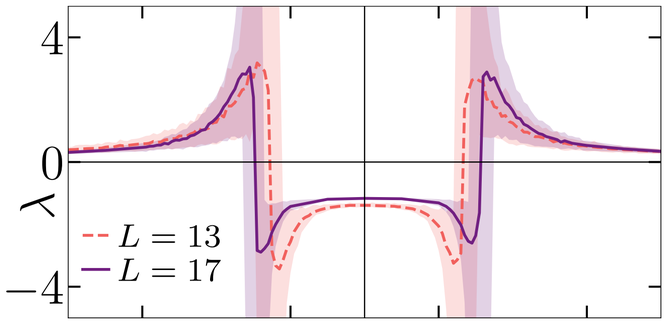
i) Many-body localised states

ii) Ergodic states

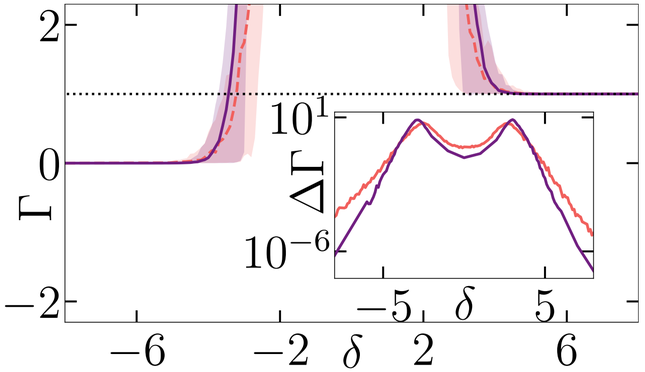
iii) Topological many-body localised states



Correlation decay length
(Localisation length)



Total info on large scales

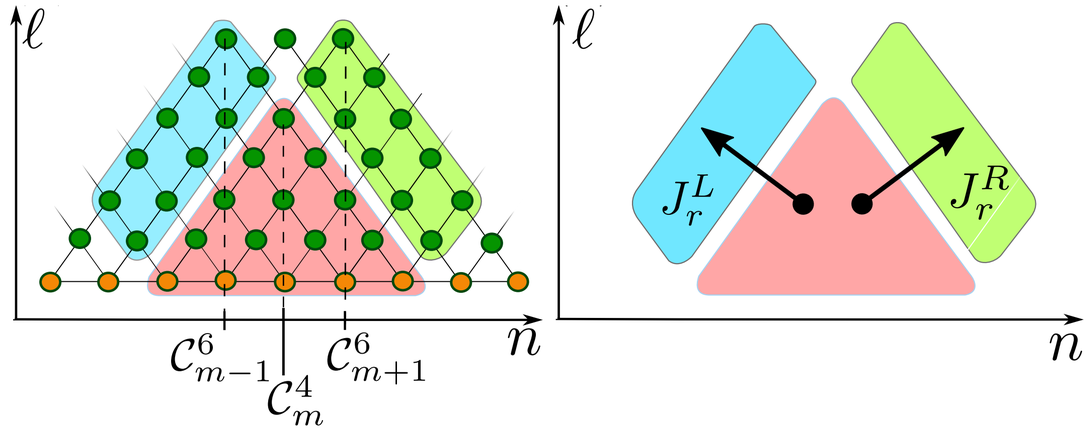


$$\Gamma = \sum_{\ell=\lfloor L/2 \rfloor}^{L-1} I_\ell = N$$

[Artiaco, Klein-Kvorning, Aceituno Chávez, Herviou, JHB, PRL 2025 (arXiv:2410.10971)]

Information locally conserved and moves locally under local unitary transformations

Total information conserved under unitary dynamics, local information i_n^ℓ is a density that is follows a local conservation law with local information currents



Technical details

$$\partial_t I(\rho_n^\ell) = -i\ \mathrm{Tr}\left(\left[\nabla S_n^\ell, H_n^\ell\right]\ \rho_n^\ell\right)$$

$$-i\ \mathrm{Tr}\left(\left[1_{2r}\otimes\nabla S_n^\ell, H_{n-r/2}^{\ell+r} - H_n^\ell\right]\ \rho_{n-r/2}^{\ell+r}\right)$$

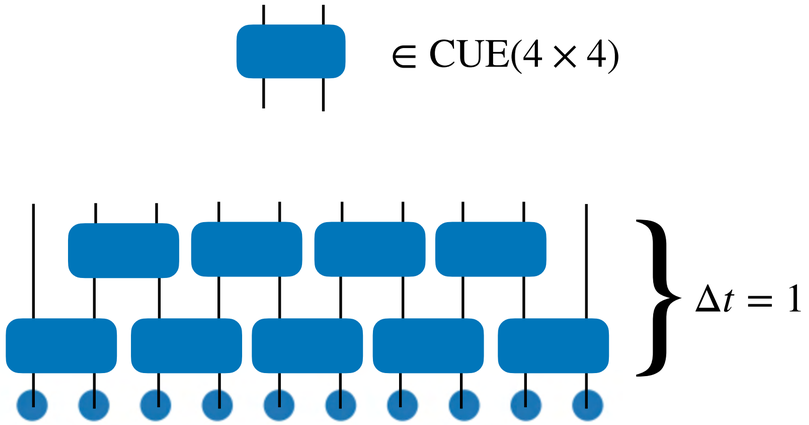
$$-i\ \mathrm{Tr}\left(\left[\nabla S_n^\ell\otimes 1_{2r}, H_{n+r/2}^{\ell+r} - H_n^\ell\right]\ \rho_{n+r/2}^{\ell+r}\right)$$

$$= J_r^R(\rho_{n-r/2}^{\ell+r}) + J_r^L(\rho_{n+r/2}^{\ell+r})$$

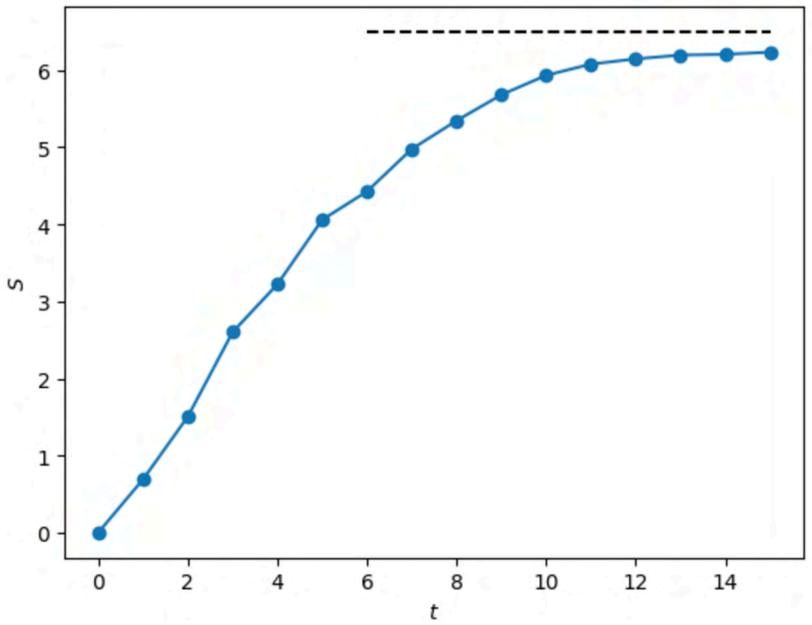
[Klein Kvorning, Herviou, JHB SciPost Physics (2022); Artiaco, Fleckenstein, Aceituno Chávez, Klein Kvorning, JHB PRX Quantum 2024]

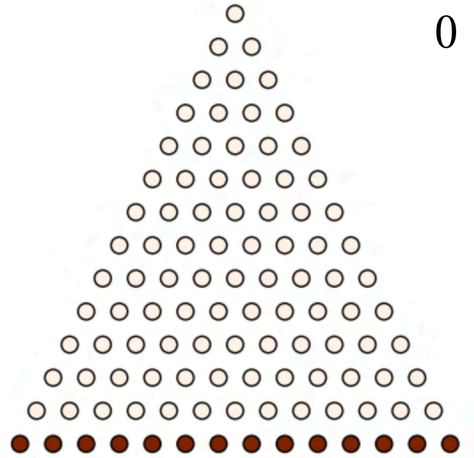
Dynamics on the information lattice – ballistic growth of entanglement in a random unitary circuit

Random brickwork of independent random unitaries

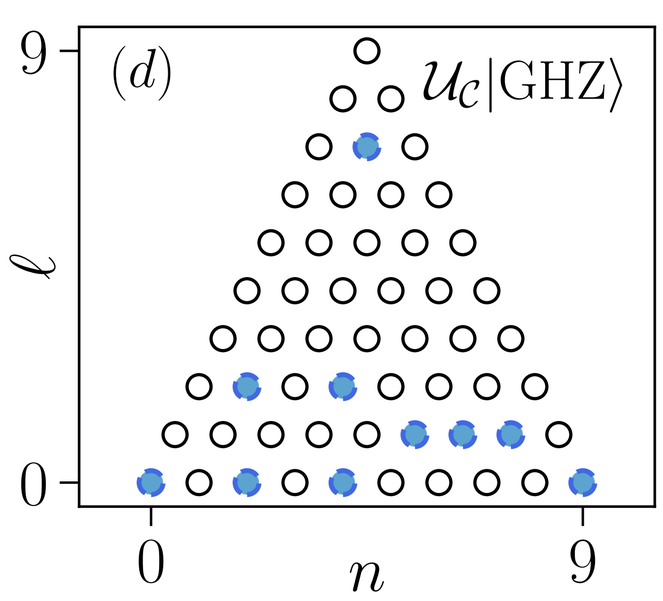


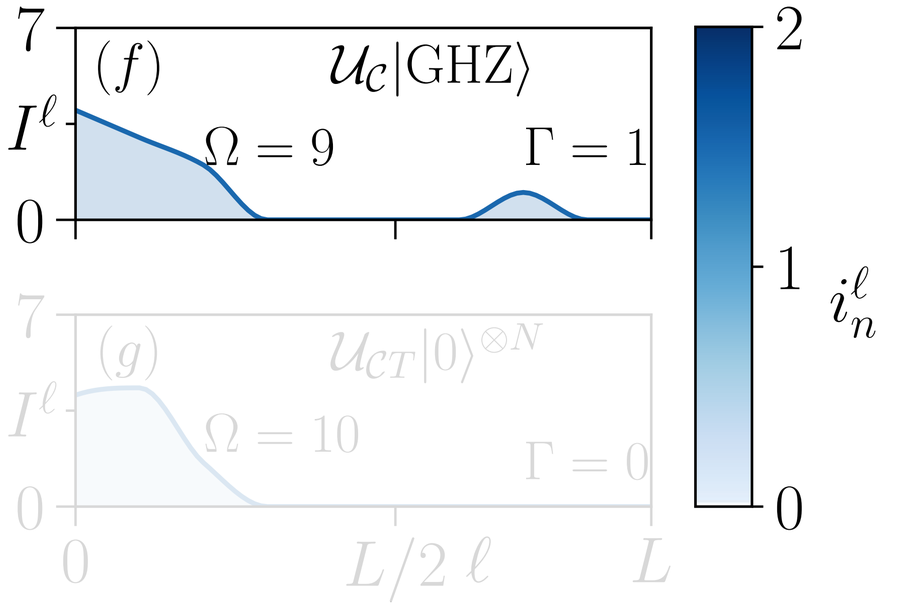
Entanglement entropy grows ballistically before saturating at the Page value





Stabilizer states with information gap and long-range magic





$\mathcal{U}_C$ Local Clifford circuit

$\mathcal{U}_{CT}$ Local Clifford+Tgate circuit

Information per scale $I^\ell = \sum_n i_n^\ell$

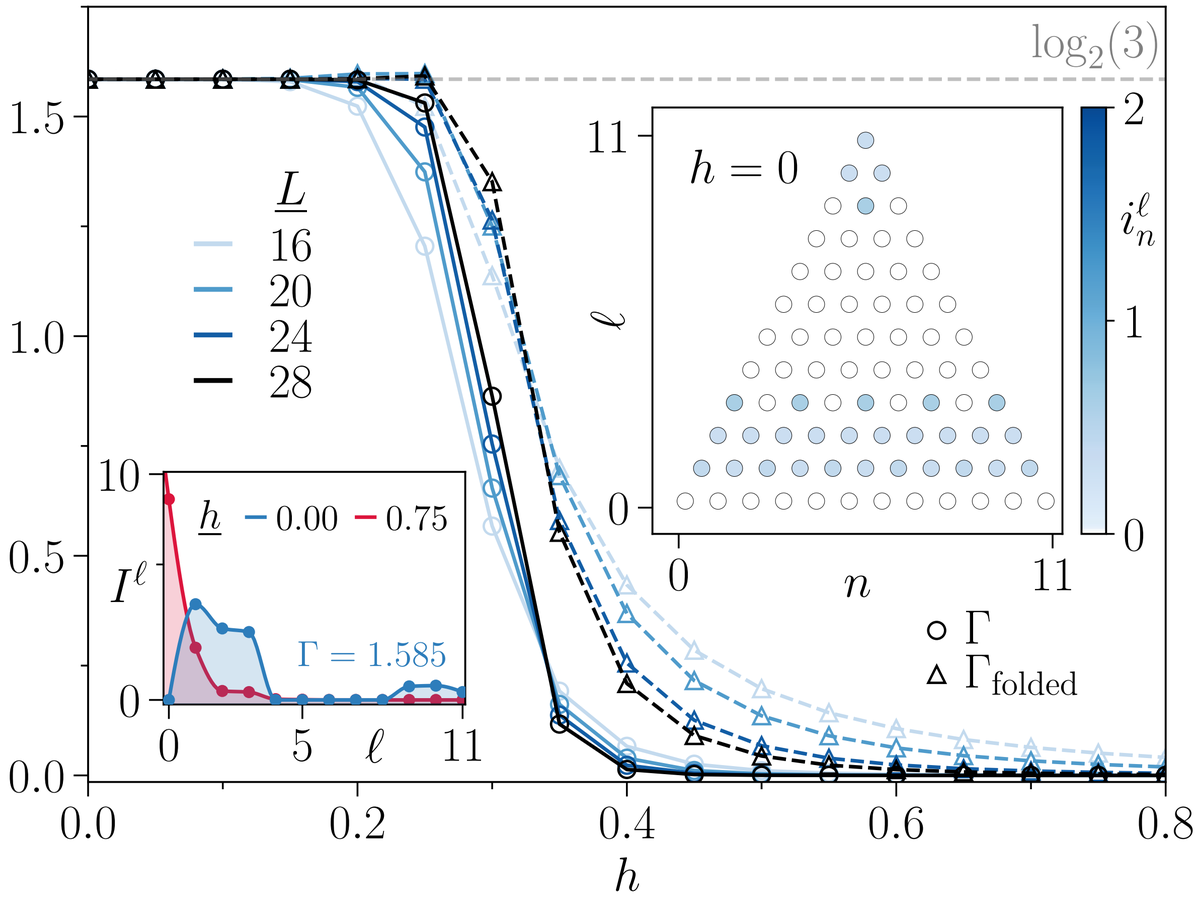
$\Gamma \notin \mathbb{N} \Rightarrow$ long-range magic

Small scale information $\Omega = \sum_{\ell=0}^{\lfloor L/2-1 \rfloor} i_n^\ell$

Large scale information $\Gamma = \sum_{\ell=\lfloor L/2 \rfloor}^{L-1} i_n^\ell$

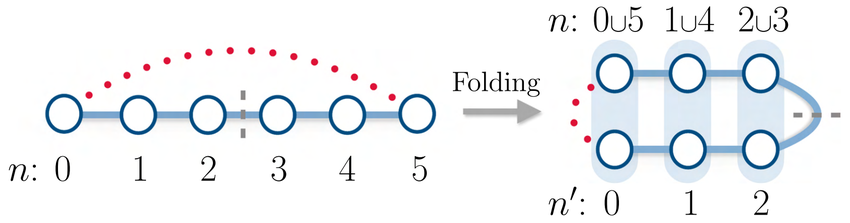
[Bilinskaya, Martínez, Ghosh, Klein Kvorning, Artiaco, JHB, arXiv:2510.26696]

Long-range magic in the three-state Potts model



$$H = -\frac{J}{3}\sum_{i}^{N-1}(Z_i^\dagger Z_{i+1} + Z_i Z_{i+1}^\dagger) - h\sum_{i}^{N}(X_i^\dagger + X_i)$$

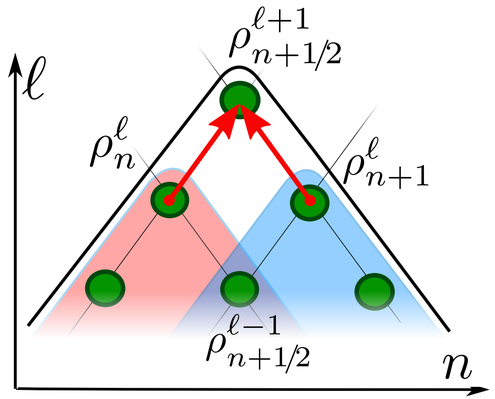
$$Z_i = \begin{pmatrix} 1 & 0 & 0 \\ 0 & \omega & 0 \\ 0 & 0 & \omega^2 \end{pmatrix} \qquad X_i = \begin{pmatrix} 0 & 0 & 1 \\ 1 & 0 & 0 \\ 0 & 1 & 0 \end{pmatrix}$$



[Bilinskaya, Martínez, Ghosh, Klein Kvorning, Artiaco, JHB, arXiv:2510.26696]

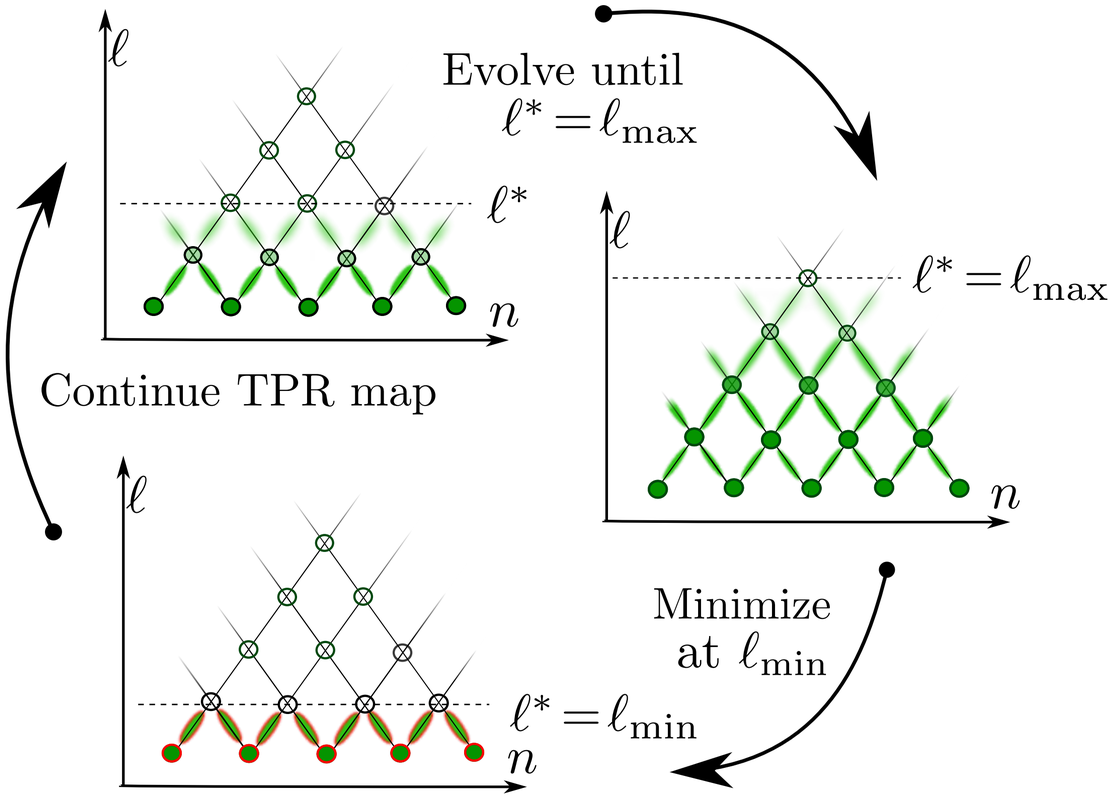
LITE – Local Information Time Evolution

If no information on scale $\ell + 1$ can construct $\rho^{\ell+1}$



$$\rho^{\ell+1}_{n+1/2} = \exp\left[\ln(\rho^{\ell}_{n}) + \ln(\rho^{\ell}_{n+1}) + \ln(\rho^{l-1}_{n+1/2})\right]$$

(Petz) recovery map



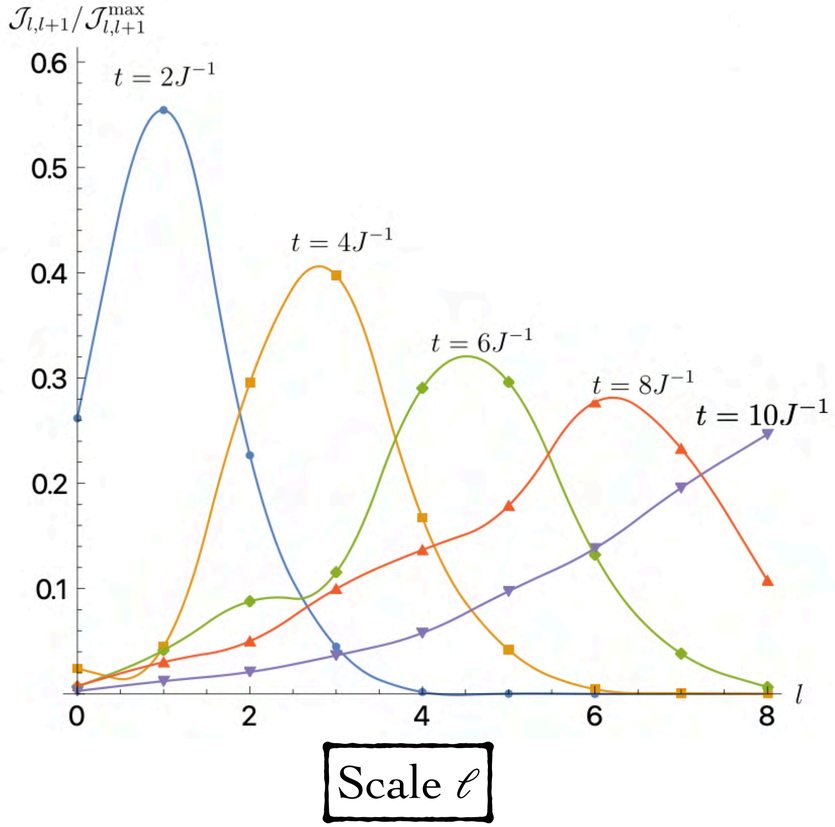
[Klein Kvorning, Herviou, JHB SciPost Physics (2022); Artiaco, Fleckenstein, Aceituno Chávez, Klein Kvorning, JHB PRX Quantum 2024]

In microscopic models information flows to large scales leaving behind a tail of information at short scales

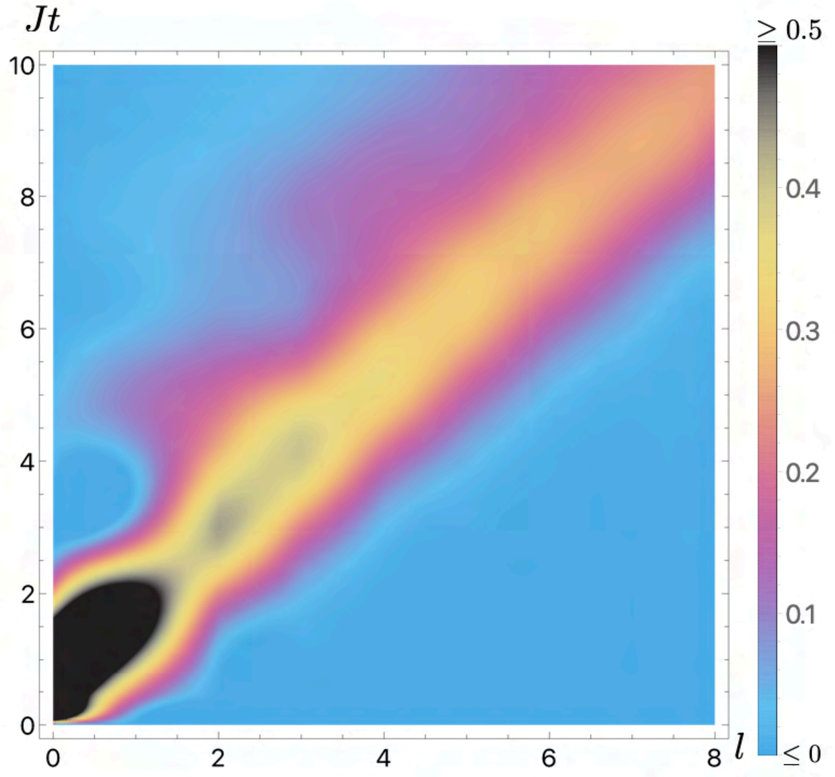
Mixed-field Ising model $H = \sum_i J\sigma_i^z\sigma_{i+1}^z + h_T\sigma_i^x + h_L\sigma_i^z$

$$\rho(t=0) = \cdots \otimes I_2 \otimes I_2 \otimes |\uparrow_x\rangle\langle\uparrow_x| \otimes I_2 \otimes I_2 \otimes \cdots$$

Total information on scale ℓ



Scale ℓ



[Klein Kvorning, Herviou, JHB SciPost Physics (2022)]

Example: energy diffusion in the mixed field Ising model

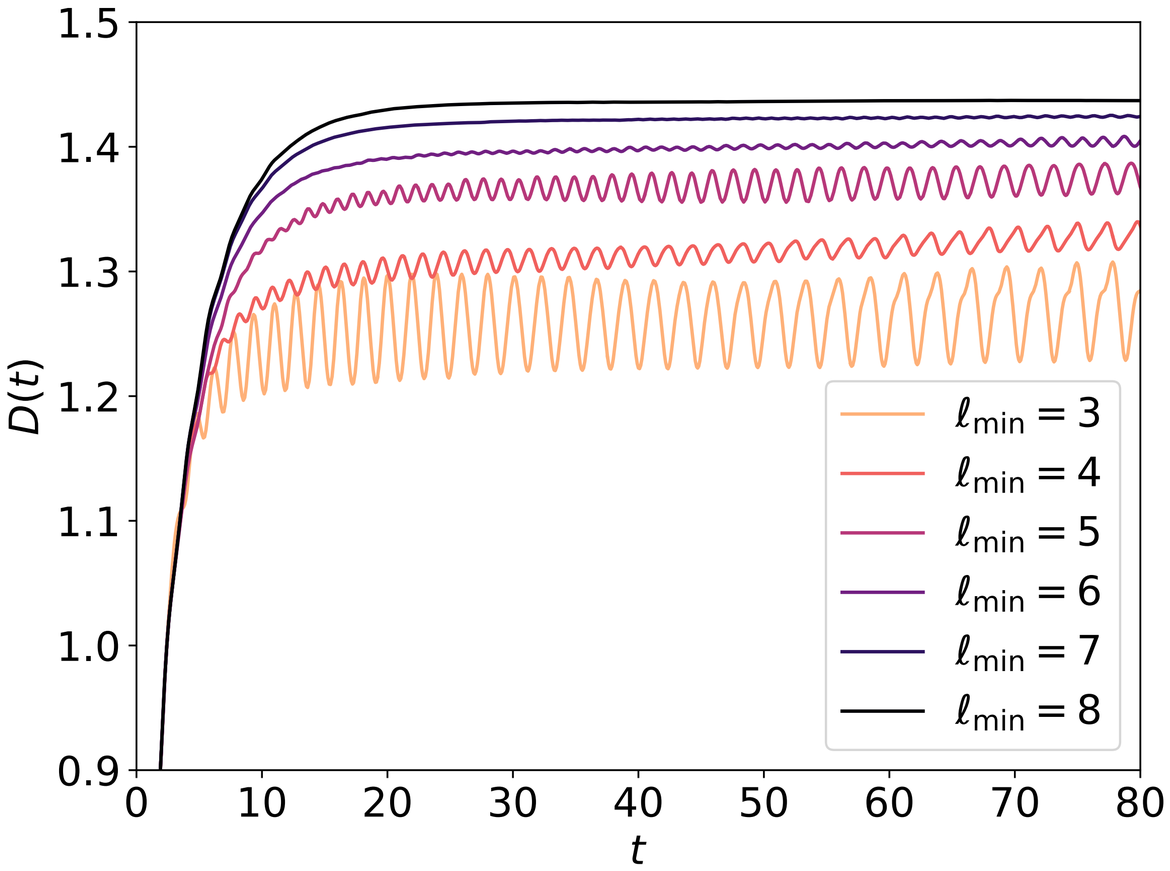
$$H = \sum_i J\sigma_i^z\sigma_{i+1}^z + h_T\sigma_i^x + h_L\sigma_i^z$$

$$J = 1,\ h_T = 1.4,\ h_L = 0.9045$$

$$\rho_{\text{init}} = \left(\bigotimes_{m<n-\ell/2} \rho_{m,\infty} \right) \otimes \rho_{n,\text{init}}^{\ell} \otimes \left(\bigotimes_{m>n+\ell/2} \rho_{m,\infty} \right)$$

$$\sigma_E^2 = \sum_n (n-\overline{n})^2 \frac{E_n^r}{\langle H \rangle} - \left(\sum_n (n-\overline{n}) \frac{E_n^r}{\langle H \rangle} \right)^2$$

$$D = \frac{1}{2}\partial_t \sigma_E^2$$



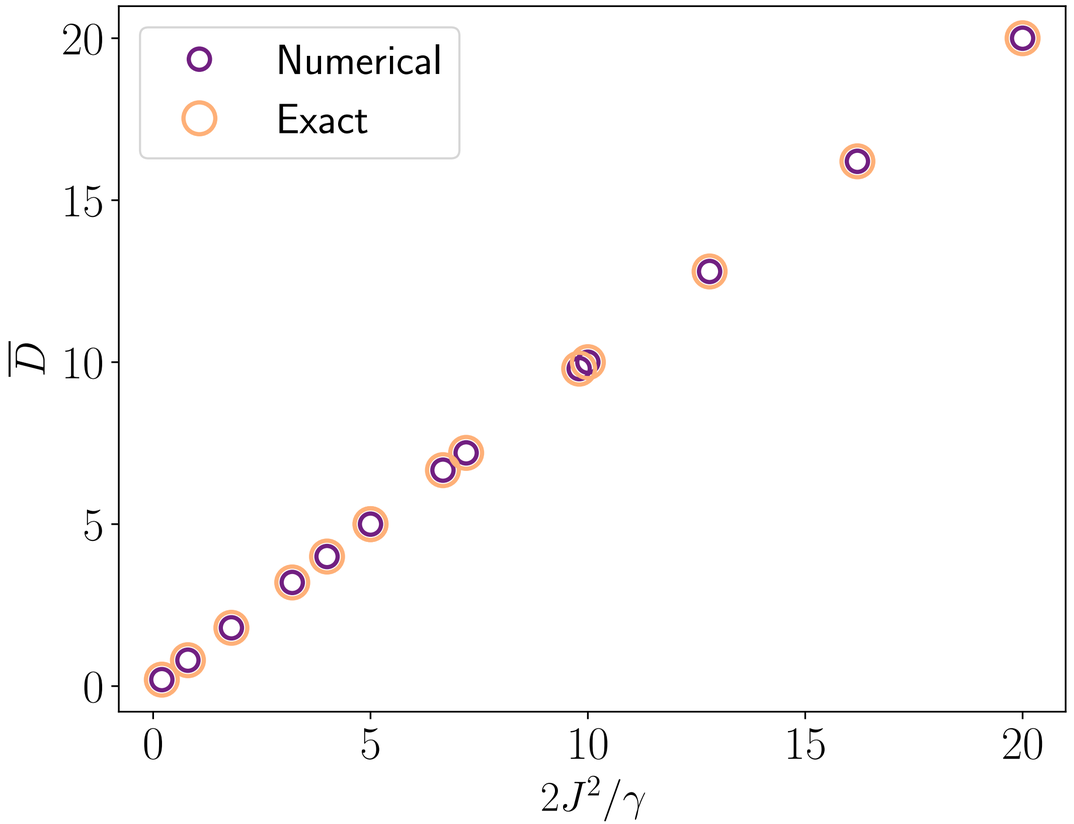
[See alos Rakovszky, von Keyserlingk, and Pollmann PRB 2022 for a different approach]

[Artiaco, Fleckenstein, Aceituno Chávez, Klein Kvorning, JHB PRX Quantum 2024]

Lindblad dynamics with local dissipators straightforwardly implemented on the information lattice



Göran Lindblad
KTH Royal Institute of Technology
9 July 1940–30 November 2022



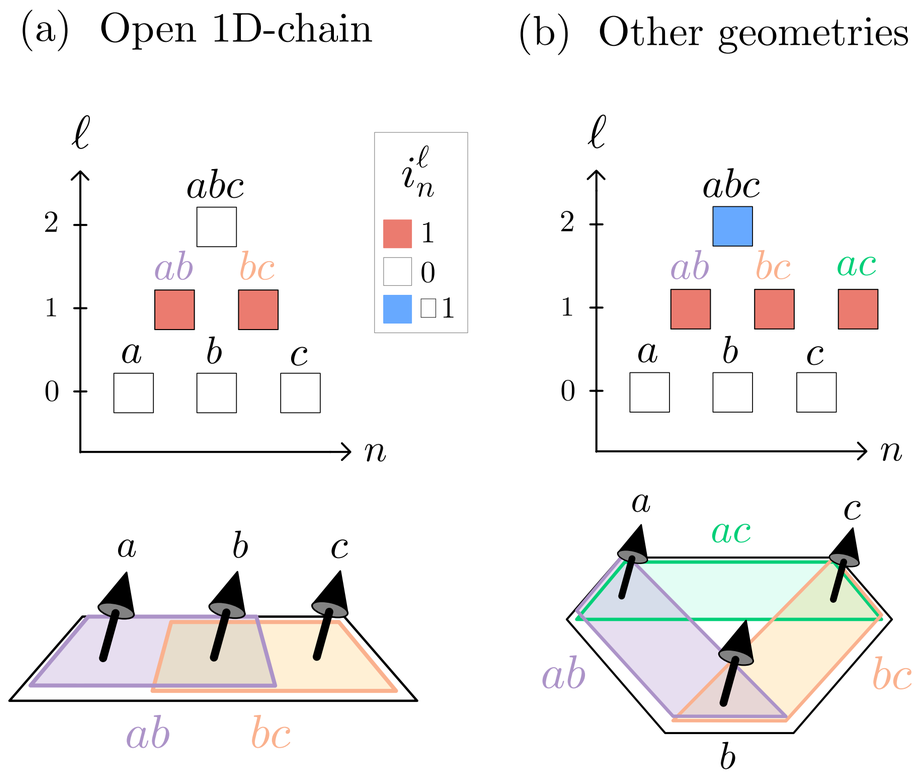
$$\partial_t \rho = -i\,[H,\rho] + \sum_j \gamma_j \left(L_j \rho L_j^\dagger - \frac{1}{2}\{L_j^\dagger L_j, \rho\} \right)$$

$$H = \sum_j J(\sigma_j^x \sigma_{j+1}^x + \sigma_j^y \sigma_{j+1}^y) \qquad L_j = \sigma_j^z; \gamma_j = \gamma \forall j$$

[Compare with Jin, Ferreira, Filipone, Giamarchi, PRR 2022]

[Artiaco, Fleckenstein, Aceituno Chávez, Klein Kvorning, JHB PRX Quantum 2024]

Generalising to higher dimensions – overlap redundancies



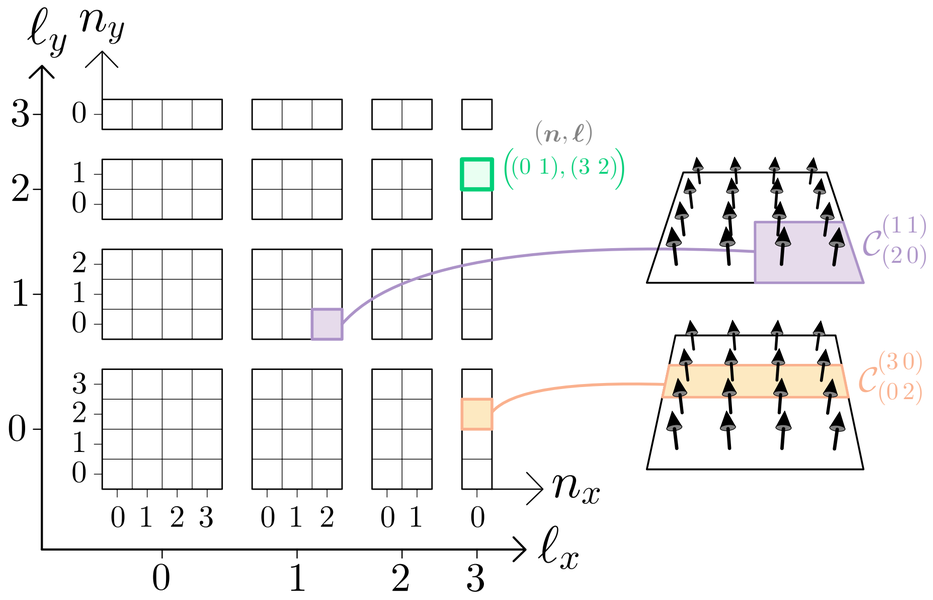
$$\text{Example state } \rho = \frac{1}{2}(|000\rangle\langle 000| + |111\rangle\langle 111|)$$

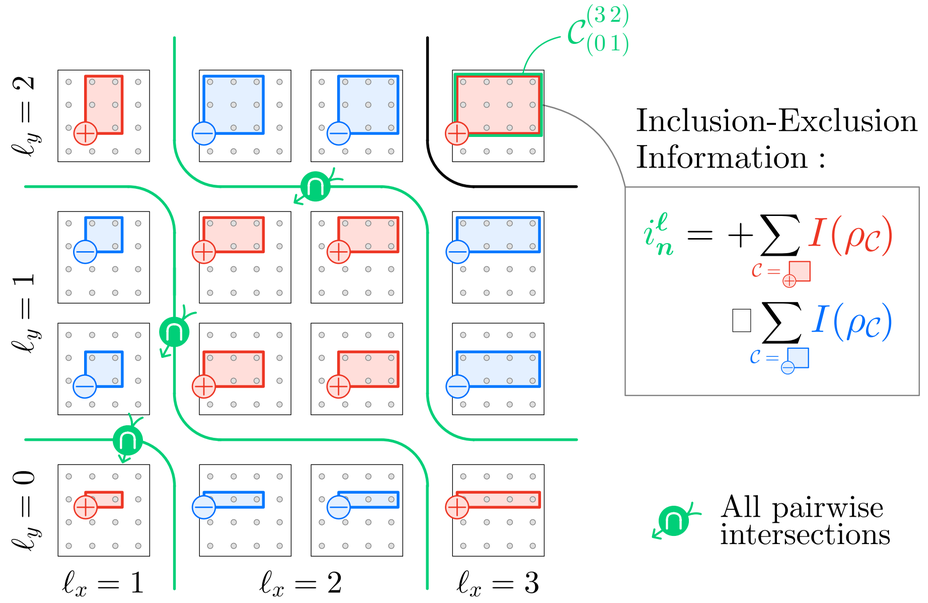
[Flór, Artiaco, Klein Kvorning, JHB, arXiv:2512:20793]

Generalising to higher dimensions – Inclusion-Exclusion information

In higher dimension there are many different choices of set of subsystems

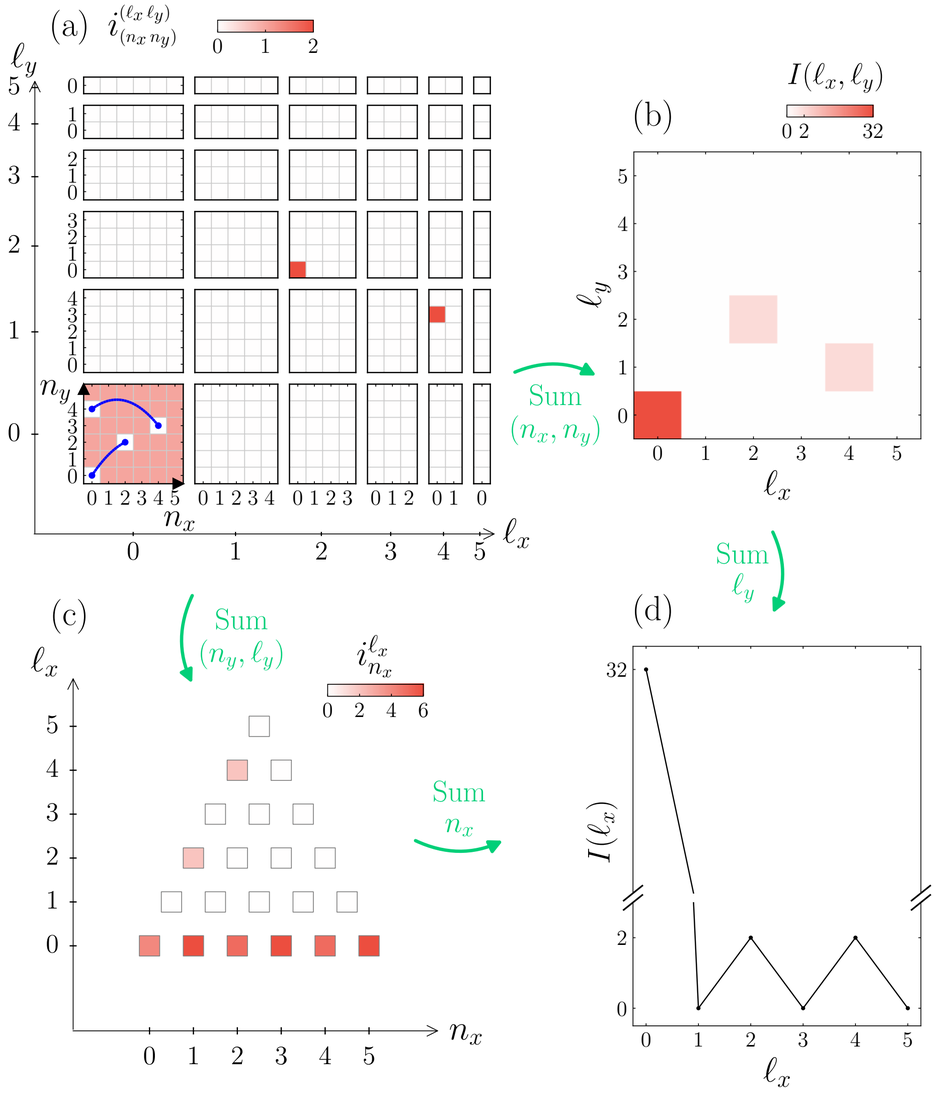
Any set that is closed under intersections allows for defining inclusion-exclusion information





[Flór, Artiaco, Klein Kvorning, JHB, arXiv:2512:20793]

Simple example of a product state with two singlets

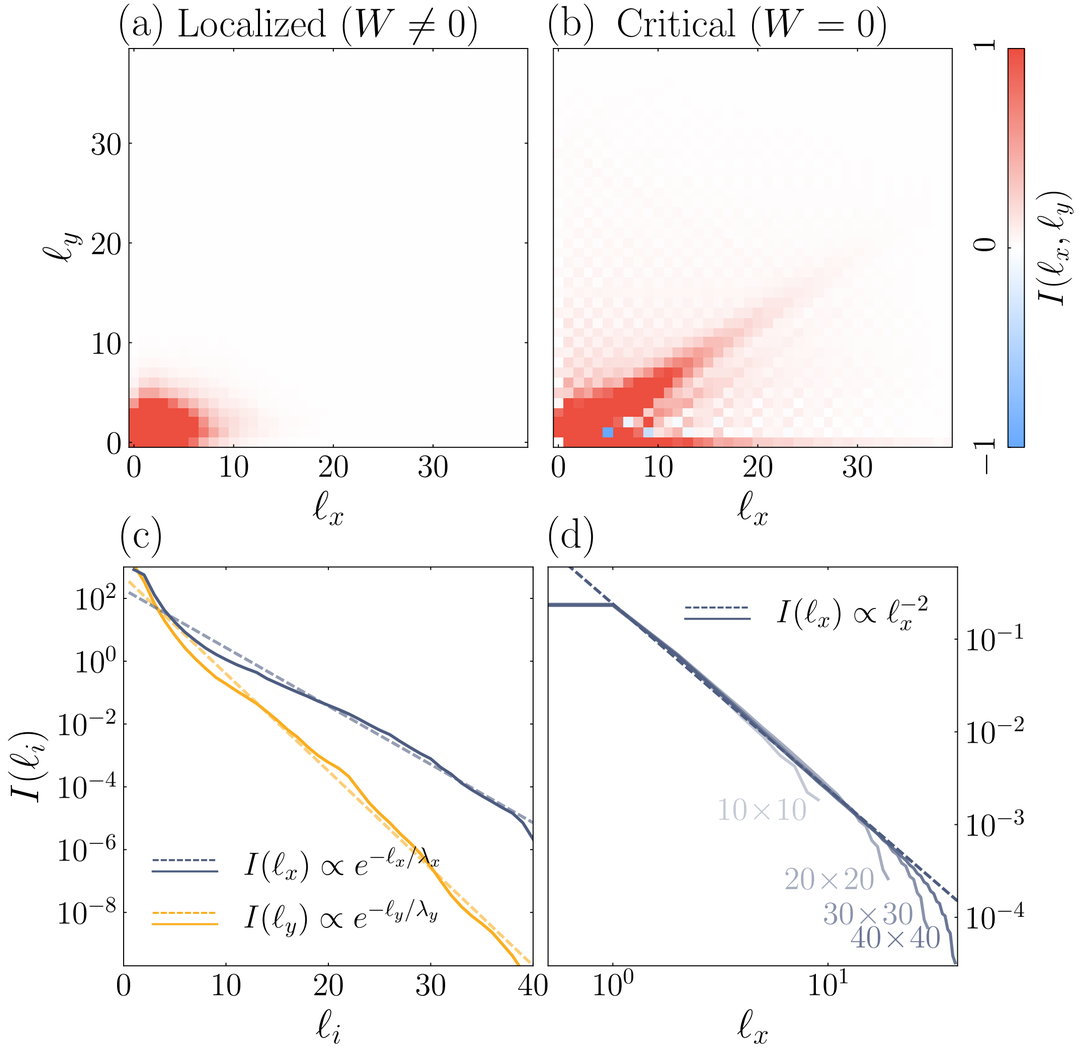


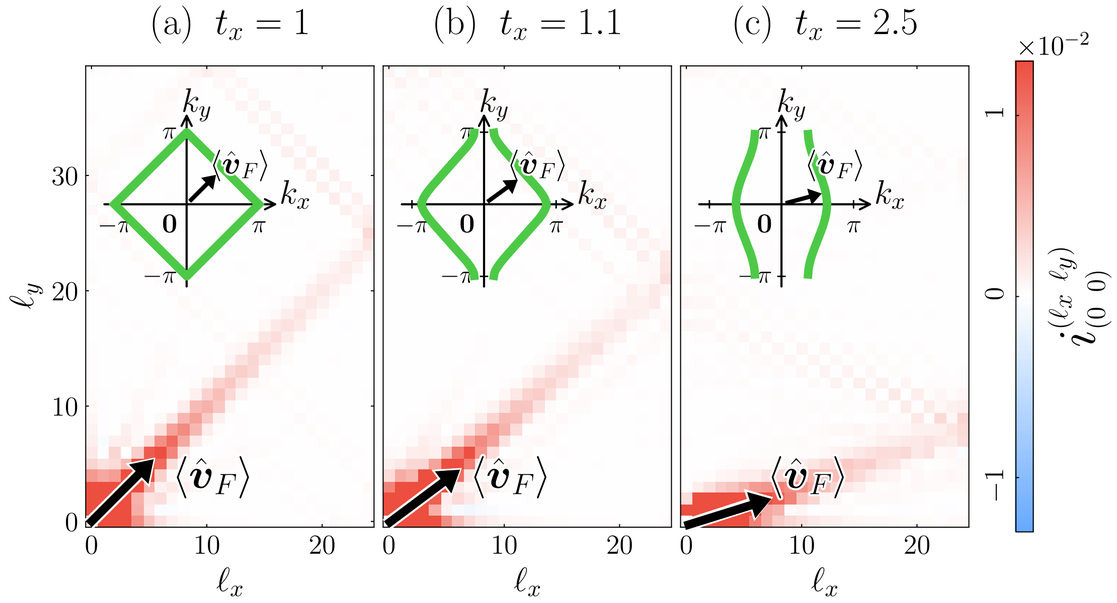
Partial summing gives quasi-1d information lattices with positive local information

Summing over one scale direction gives direction dependent information per scale

[Flór, Artiaco, Klein Kvorning, JHB, arXiv:2512:20793]

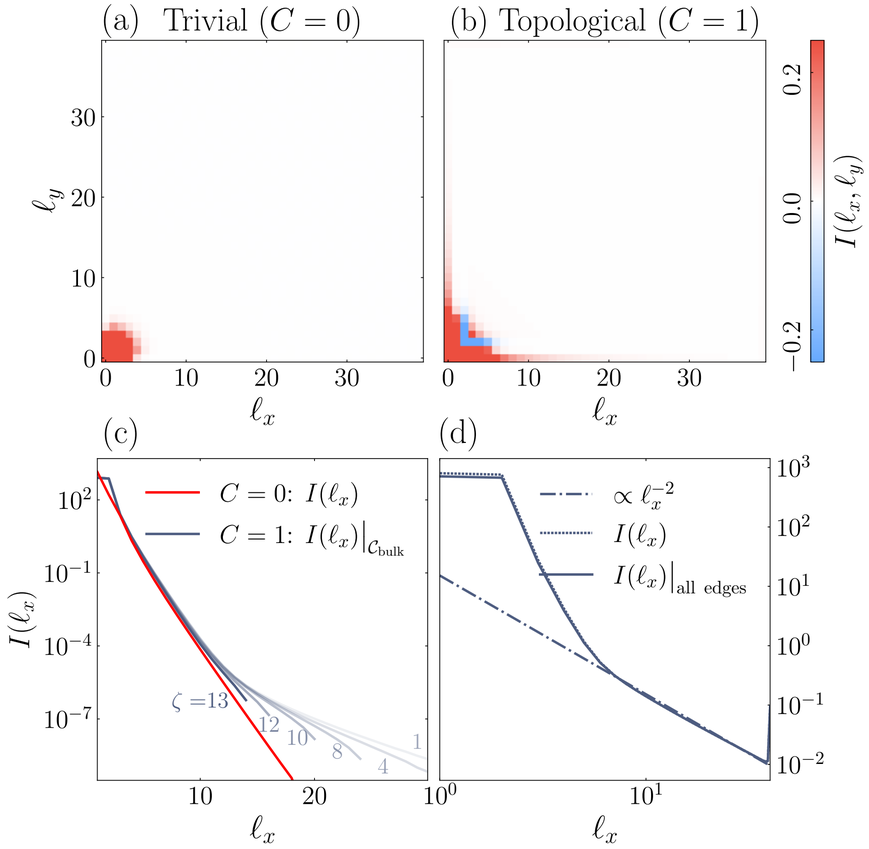
2D Anderson model with anisotropic hopping

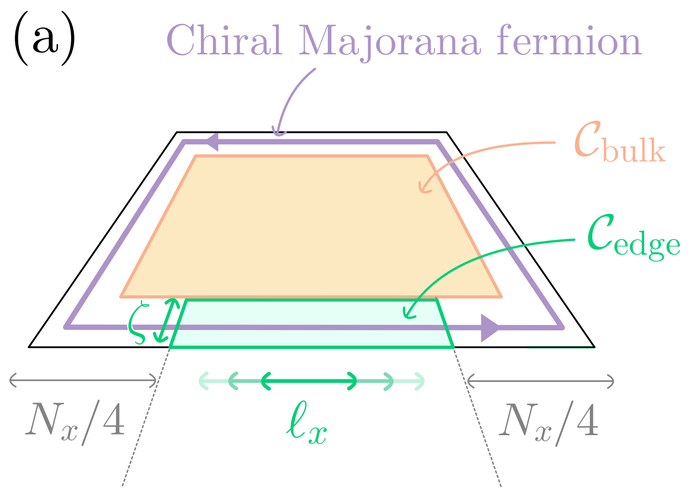


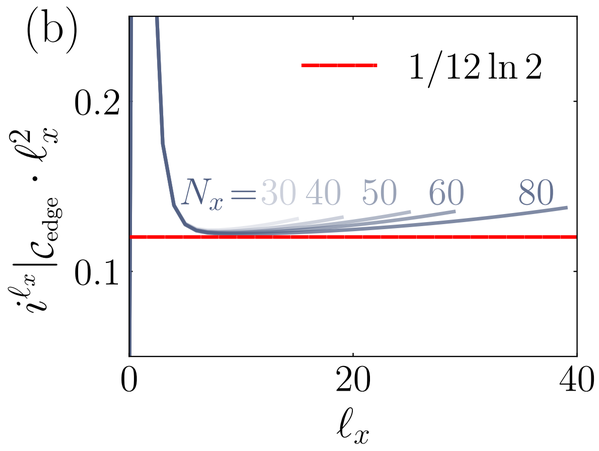


[Flór, Artiaco, Klein Kvorning, JHB, arXiv:2512:20793]

$p + ip$ superconductor





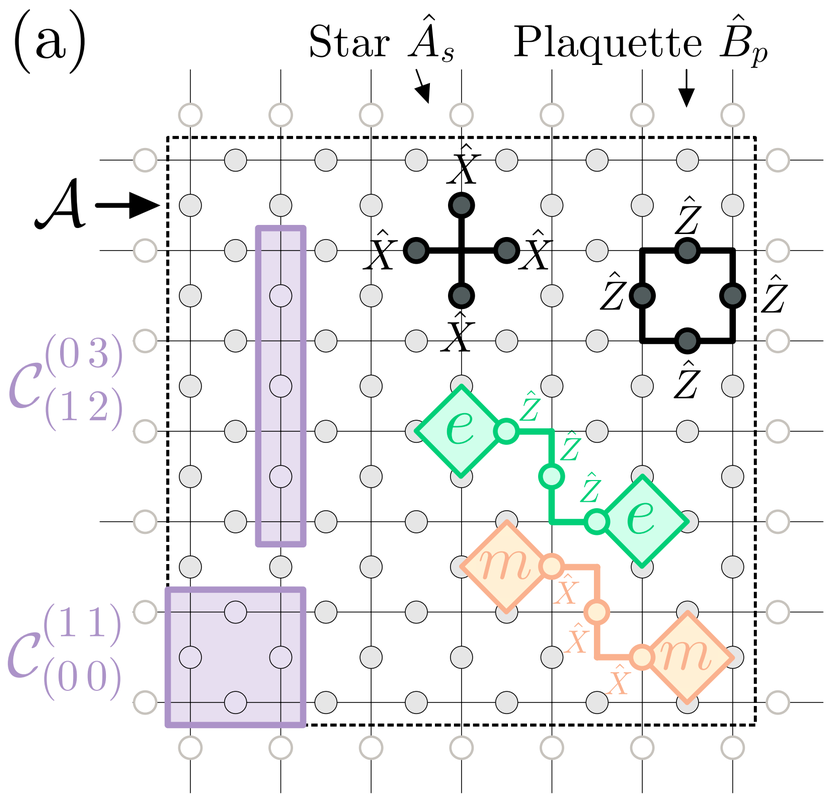


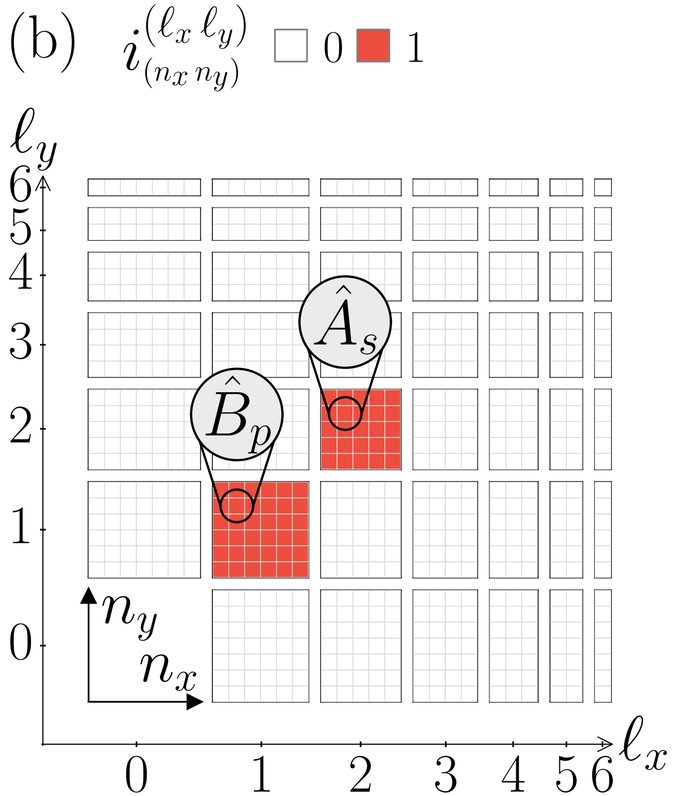
Can separate information in bulk and at boundary

Bulk exponentially decaying, edge follows a power law with appropriate central charge

[Flór, Artiaco, Klein Kvorning, JHB, arXiv:2512:20793]

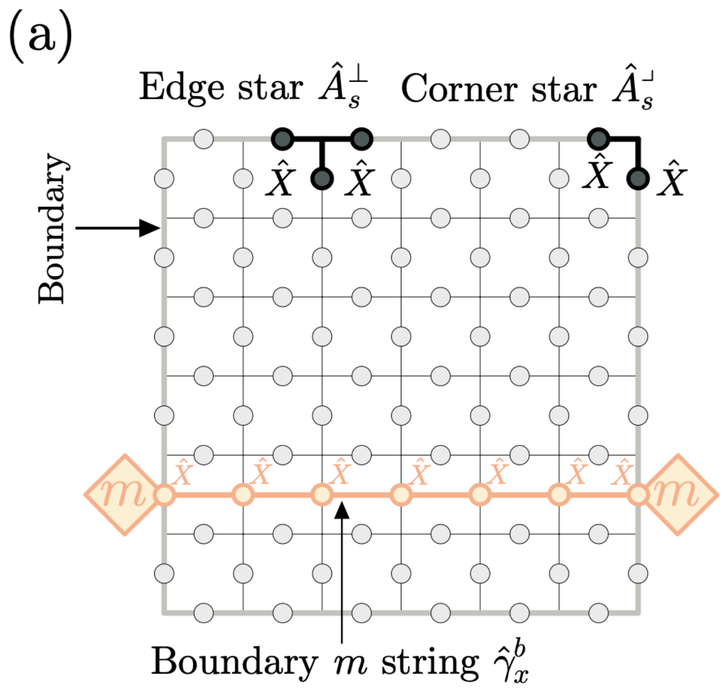
Toric code – bulk information is local

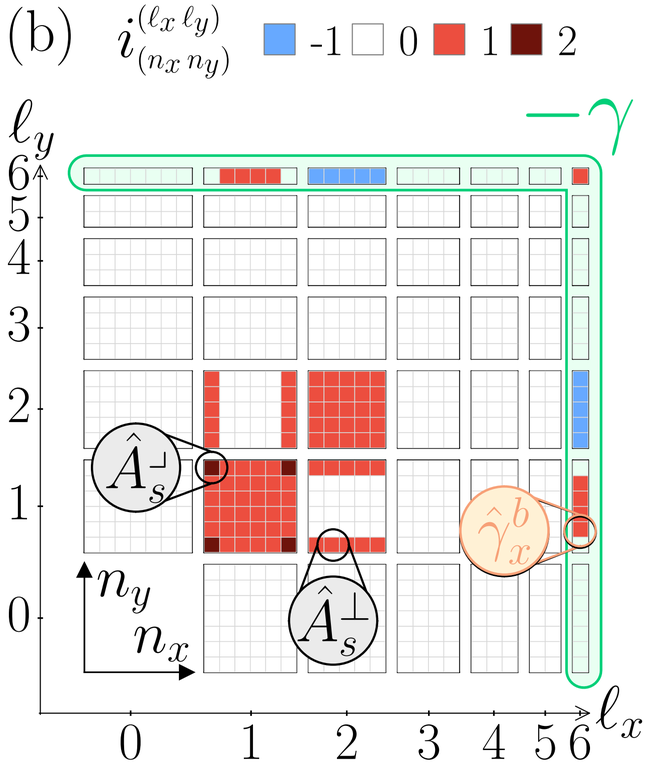




[Flór, Artiaco, Klein Kvorning, JHB, arXiv:2512:20793]

Toric code – topological entanglement is an overlap redundancy





Topological entanglement entropy γ arises from redundancies of strings

Can not be changed by local transformations

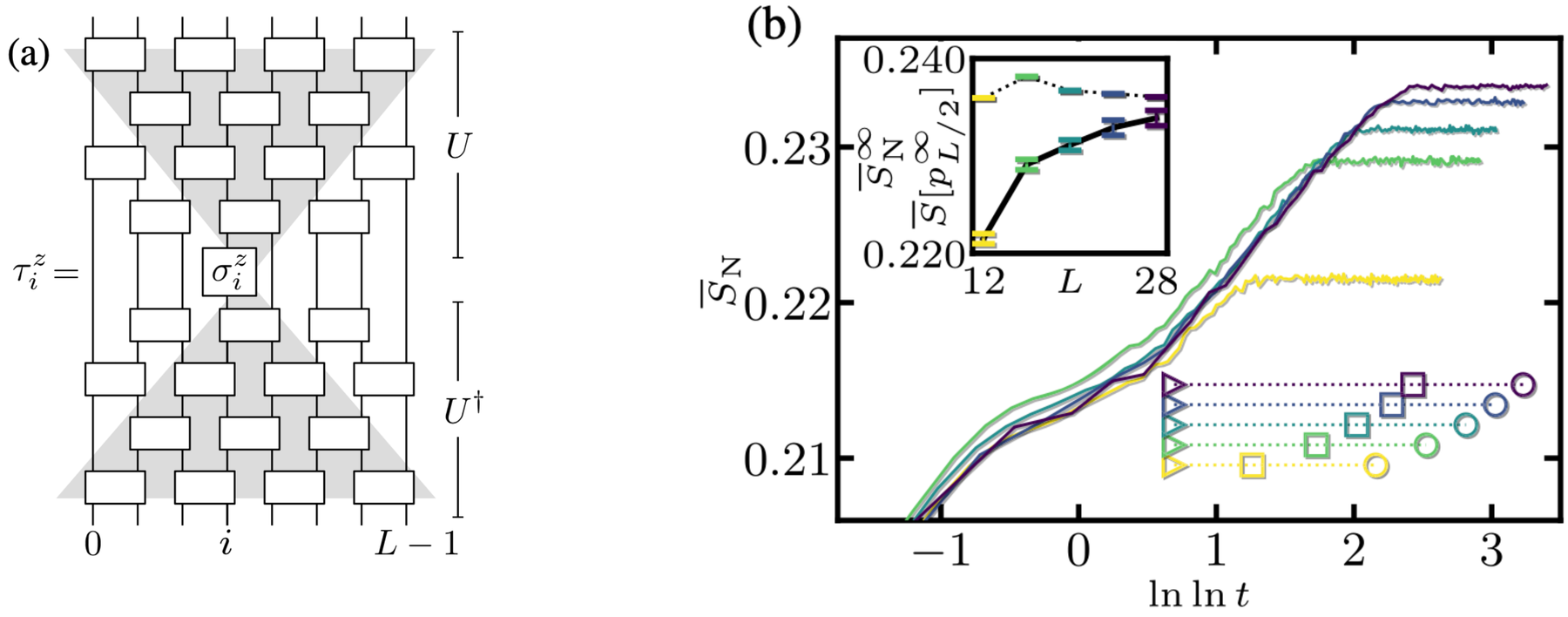
[Flór, Artiaco, Klein Kvorning, JHB, arXiv:2512:20793]

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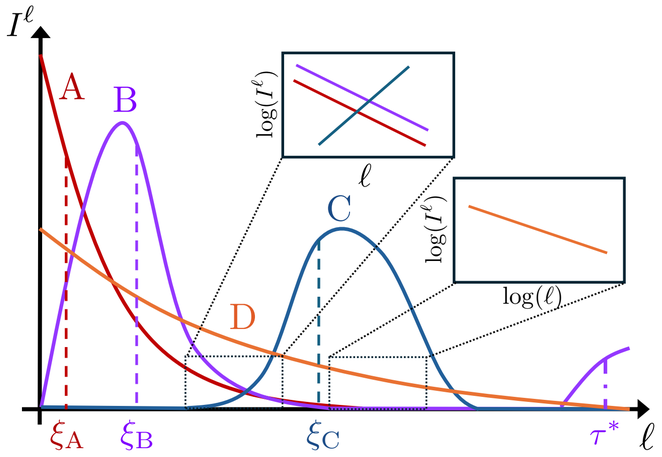
PHYSICAL REVIEW LETTERS **133,** 126502 (2024)

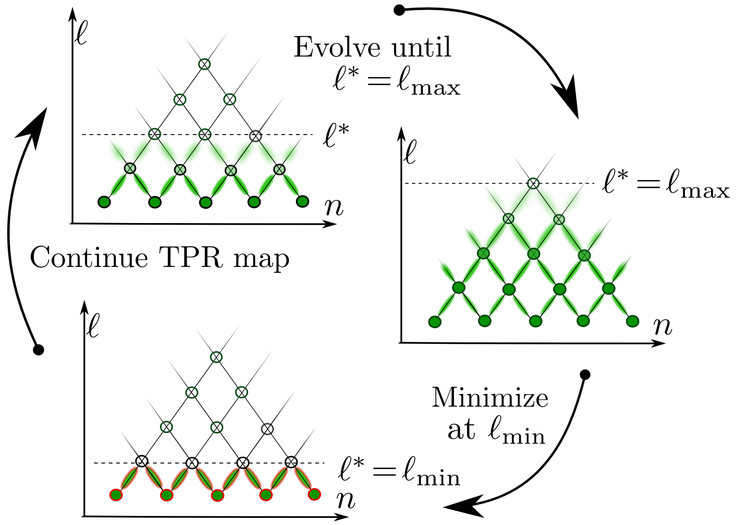
Ultraslow Growth of Number Entropy in an ℓ-Bit Model of Many-Body Localization

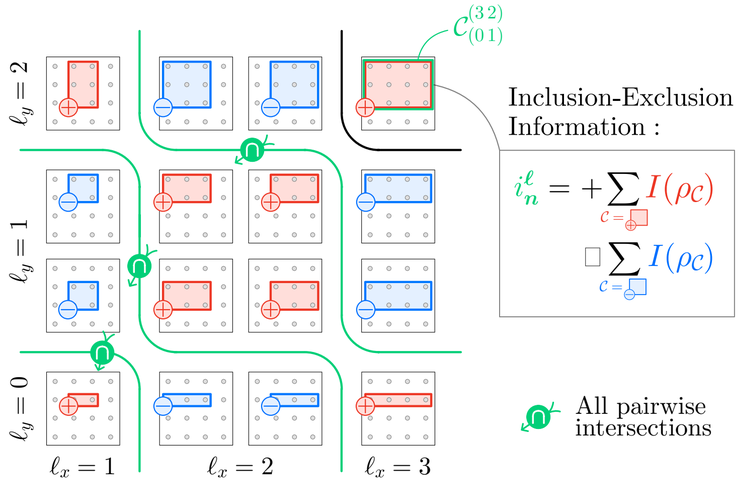
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Summary







SciPost Physics **13**, 080 (2022); PRX Quantum **5**, 020352 (2024); PRL **134**, 190401 (2025); arXiv:2510.26696; arXiv:2512:20793